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Annual Report

For

RELIABILITY AND SURVIVABILITY OF COMMUNICATION NETWORKS

Contract No. N00014-86-K-0745

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RE: Annual Letter Report

Principal Investigator:	Peter J. Slater
Contract Name:	FONR Comm Networks
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PUBLICATIONS (2 and 3 are updated from the 1987 report.):

2. Exact Formulas For Reliability Measures For Various Classes Of Graphs, Congressus Numerantium 58, 1987, pp.43-52.

3. Pair-Connected Reliability Of A Tree and Its Distance Degree Sequence, Congressus Numerantium 58, 1987, pp. 29-42.

4. On The Non-Existence Of Uniformly Optimal Graphs For Pair-Connected Reliability, submitted for publication.

5. A Summary Of Results On Pair-Connected Reliability, to appear in the Proceedings of the NSF Conference on Graphs and Algorithms.

6. Pair-Connected Reliability Of Communication Networks With Vertex Failures, to appear in the Proceedings of the 19th Southeastern Conference on Combinatorics, Graph Theory and Computing.

7. The Distribution Of Pair-Connectivity For Network Reliability, in preparation.

8. Pair-Connected Reliability With Vertex Failures: Uniform Optimality, in preparation.

PRESENTATIONS:

5. (Reported last year) Dr. Slater presented an invited lecture, "The Pair-Connected Measure Of Network Reliability", at the Graphs and Algorithms Conference in Boulder, Colorado, June, 1987.

6. Dr. Slater presented "Cutting Numbers For Series-Parallel Graphs" at the TIMS/ORSA Joint National Meeting, 1987.

7. Dr. Slater presented 6 (above) at the 19th Southeastern Conference, February, 1988.

8. Dr. Siegrist presented 7 (above) at the 19th Southeastern Conference, February, 1988.

9. Dr. Slater presented an invited address, "Pair-Connected Network Reliability", at the First Cumberland Conference on Graph Algorithms and Combinatorics, Tennessee Tech University, April, 1988.

SUMMARY REPORT:

This is the second annual report on the study of network reliability, specifically pair-connected reliability. We assume we have a probabilistic graph G, a graph with n vertices and m edges, representing a communications network with each vertex representing a processor and each edge a communications link. As in the first annual report, we use "PC" to denote "pair-connectivity" parameters. Here, let PC (G;p) be the expected number of pairs of vertices that are connected when each of the vertices is fail-safe, but each edge is independently in a failed state with probability q. So $p = 1 - q$ is the probability that each edge is operable. In general, we can write PC (G;p) as

$$PC(G;p) = \sum_{i=1}^{m} A_i p^i$$

I. In publication [4], we examine how the coefficients A_i of the pair-connected reliability polynomial PC(G;p) are determined by the subgraph structure of G. While evaluating PC(G;p) is in general an NP complete problem, we have the following.

<u>Theorem A</u> For a fixed value k, the coefficients $A_1, \ldots, A_k$ can be computed in time polynomial in n.

For the "global reliability" measure (the probability that G remains connected) Boesch's conjecture is that for each choice of n and m there exists a uniformly optimal (n,m)-graph G, one which for all p with $0 < p < 1$ is at least as reliable as any other (n,m)-graph. A principal result of [4] is the following.

Theorem B With respect to pair-connected reliability, there does not exist a uniformly optimal (n,m)-graph if

$$n \leq m \leq \binom{n}{2} - 2.$$

While the most widely studied model for network reliability is the one in which vertices do not fail and each edge fails independently with probability q, in [6] we consider the case in with edges are fail-safe but each vertex is in a failed state with probability q= 1-p. Among the results for a vertex failure model which are similar to those for edge failures is a result like Theorem A. However, a "Theorem B for vertex failures" is false. Nor is there always a uniformly optimal graph. We are currently trying to determine for precisely which values of m do there exist uniformly optimal (n,m)-graphs.

II. As noted earlier, the pair-connected reliability of a graph is the expected value of the random variable which gives the number of pairs of connected vertices when the edges (or vertices) are failure prone. Thus, the pair-connected reliability is but one measure of the center of the distribution of the underlying random variable and should not be studied in isolation. For example, some indication of the spread of the distribution with respect to the expected value is essential as a measure of the quality of the pair-connected reliability measure. Another essential consideration is the general shape of the distribution, particularly for large graphs. We are currently studying both of these problems using the theory of martingale difference sequences.

The importance of studying the entire distribution, rather than just its expected value, is strikingly clear in the vertex-failure model. For example, it is easy to show that the star on n vertices is optimal in the class of trees on n vertices with respect to pair-connected reliability. However, it is also clear that if the center vertex of the star fails, then the graph becomes totally disconnected - obviously a very undesirable quality. Indeed, it would be easy to define reasonable measures of reliability (such as the median or mode of the distribution) in which the star would be one of the worst trees.

The variance of the pair-connectivity random variable can be computed for a specific graph in much the same way that the pair-connected reliability itself can be computed. However, to obtain general information about the variance for a class of graphs, in terms of the parameters of the graph, is extremely difficult. For example, the analysis of how the variance of the random variable for the path on n vertices (with edge failures) grows with n (surely one of the simplest possible problems of this sort) turns out to be surprisingly difficult and tedious. (The answer, by the way, is np/q^3 + lower order terms.) The problem for more complicated classes of graphs seems completely intractible. We are using an alternate approach based on martingale difference sequences to obtain a different measure of the dispersion of the distribution. This measure is generally better than the variance and seems much easier to compute. We have obtained quite explicit results for trees, and we are currently trying to extend the technique to other classes of graphs.

Our study of the shape of the distribution when the graph is large has concentrated on central limit type of results. Using martingale difference sequences we have obtained general conditions which are sufficient for the asymptotic normality of the distribution as the number of vertices grows large. We have verified that these conditions hold for graphs which are sufficiently star-like, but we believe that the conditions are actually satisfied for very general graphs and we hope to make additional progress in this area.

III. Much of the current effort is involved in computing PC(G;p). Recently the Alabama Supercomputer (a CRAY-XMP/24) became operational (Spring, 1988), and we have modified our existing codes for use on the CRAY. (In particular we have extended our computer code to allow the computation of the variable and the distribution of the pair-connectivity random variable.) Even for moderately sized networks, computation of PC(G;p) is extremely time consuming. (One (26, 52)-graph required more than nine hours of CRAY cpu-time for the vertex failure problem.) Efficient approximation and exact techniques are being studied for PC in the design and analysis of communication networks.